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Modeling fully triangular neutrosophic linear programing problem and its solution algorithm

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Index Terms
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Abstract
This study attempts to establish a new model of linear fractional programming LFP problem, where the information on decision-making units is triangular neutrosophic number (TNN). A new method based on the ranking function to find the neutrosophic optimal solution of fully triangular neutrosophic linear programming (FTNLP) problem is proposed. In addition, the model is compared with existing methods. Finally, a numerical example is given to show that the method is computationally very simple and comprehensible.

نمذجة مسألة البرمجة الخطية النيوتروسوفيكية المثلثية بالكامل وخوارزمية حلها

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الكلمات المفتاحية
البرمجة الكسرية الخطية، المجموعات النيوتروسوفية، الأعداد النيوتروسوفية المثلثية ذات القيم المفردة.

الملخص
تم في هذه الدراسة إنشاء نموذج جديد لمشكلة البرمجة الكسرية الخطية (**LFP**)، حيث تكون المعلومات عن وحدات اتخاذ القرار على شكل أعداد نيوتروسوفية مثلثية (**TNN**)، وذلك باقتراح طريقة جديدة تعتمد على دالة الترتيب لإيجاد الحل الأمثل النيوتروسوفي لمشكلة البرمجة الخطية النيوتروسوفية المثلثية بالكامل (**FTNLP**). بالإضافة إلى ذلك، تم مقارنة النموذج مع الطرق الموجودة. وأخيرًا، تم تقديم مثال رقمي لإظهار أن الطريقة بسيطة جدًا من الناحية الحسابية وسهلة الفهم.

1. INTRODUCTION

The linear fractional programming (LFP) problem has significant applications in various areas of life such as production planning, financial sector, health care, and all engineering fields. The main aim of this problem is maximize/minimize a ratio of physical and/or economical linear functions subject to linear constraints. several methods (see [6, 25]) have been recommended to solve linear fractional programming (LFP) problems. In the actual applications, a model involves many parameters whose values are given by experts. However, both experts and decision makers frequently do not know the value of those parameters due to various uncontrollable factors. From this point of view, many researchers [5, 12, 20, 27] dealt with the imprecision of parameters by using the concept of fuzzy set (FS) theory [7, 30] and represented some or all of the parameters as fuzzy numbers. So, the linear fractional programming is generalized to fuzzy linear fractional programming (FLFP). Although the theory of intuitionistic fuzzy sets (IFSs) can handle incomplete information in various real-world issues, it cannot address all types of uncertainty, such as indeterminate information. Therefore, Smarandache [22, 24, 21] presented the neutrosophic set (NS) as a robust tool that generalizes the classical and all kinds of FSs (FSs and IFSs). NSs can accommodate ambiguous, indeterminate, and conflicting information where the indeterminacy is clearly quantified and define three independent degrees namely truth-membership degree (T), indeterminacy-membership degree(I), and falsity membership degree (F). The decision makers in neutrosophic set want to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership. In the past few years, the field of NS, logic, measure, probability, and statistics, pre-calculus, and calculus, and their applications in multiple areas have been extended and applied in various fields. For more information, see [4, 8, 9, 11, 13, 15, 16, 17, 18, 19, 26, 28, 29]. This paper attempts to formulate and solve the linear fractional programming (LFP) problem in a neutrosophic environment. The problem is considered by incorporating for single valued triangular neutrosophic number. A new method to find the neutrosophic optimal solution of FTNLFP problem is proposed.

This paper is organized as follows: In Section 2 some basic definitions and arithmetical operations with respect to the neutrosophic number are presented. In Section 3, fully triangular neutrosophic linear fractional programming (FTNLFP) problem is formulated and a solution method for obtaining the solution is introduced. In section 4, the proposed system applied with representation numerical guide to explain its appropriateness. Finally, some concluding remarks are reported in Section 5.

2. PRELIMINARIES

Definition 1. [23]
Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, $T_A(x)$, $I_A(x)$ and $F_A(x)$, are real standard or real nonstandard subsets of $]0^-,1^+[$. That is $T_A(x):X \longrightarrow]0^-,1^+[$, $I_A(x):X \longrightarrow]0^-,1^+[$ and $F_A(x):X \longrightarrow]0^-,1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq supT_A(x) \leq supI_A(x) \leq supF(x) \leq 3^+$.

Definition 2. [1, 2]
Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form $A = \{x, T_A(x), I_A(x), F_A(x)\}$, where $T_A(x):X \longrightarrow [0,1]$, $I_A(x):X \longrightarrow [0,1]$ and $F_A(x):X \longrightarrow [0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$, for all $x \in X$.

Definition 3. [10]
A triangular neutrosophic number (TNN) is signified via $\tilde{A}^N = \langle (a_1, a_2, a_3; \mu_A, \omega_A, \varphi_A) \rangle$ is an extended version of the three membership functions for the truth, indeterminacy, and falsity of x can be defined as follows:

$$T_A(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & a_1 \leq x < a_2 \\ 1 & x = a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & a_2 \leq x < a_3 \\ 0 & something\ else \end{cases}$$

$$I_A(x) = \begin{cases} \frac{(a_2-x)}{(a_2-a_1)} & a_1 \leq x < a_2 \\ 1 & x = a_2 \\ \frac{(x-a_2)}{(a_3-a_2)} & a_2 \leq x < a_3 \\ 0 & something\ else \end{cases}$$

$$F_A(x) = \begin{cases} \frac{(a_2-x)}{(a_2-a_1)} & a_1 \leq x \leq a_2 \\ 0 & x = a_2 \\ \frac{(x-a_2)}{(a_3-a_2)} & a_2 \leq x \leq a_3 \\ 1 & something\ else \end{cases}$$

Where, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$, $x \in A$. Additionally, when $a_1 \geq 0$, A is called a nonnegative TNN. Similarly, when $a_1 < 0$, A becomes a negative TNN.

Definition 4. [3]. (Arithmetic Operations)
Suppose $\tilde{A}^N = \langle (a_1, a_2, a_3; \mu_A, \omega_A, \varphi_A) \rangle$ and $\tilde{B}^N = \langle (b_1, b_2, b_3; \mu_B, \omega_B, \varphi_B) \rangle$ be two TNNs. Then the arithmetic relations are defined as:

1) $\tilde{A}^N \oplus \tilde{B}^N = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle$

.

2) $\tilde{A}^N \ominus \tilde{B}^N = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle$

.

3) $\tilde{A}^N \otimes \tilde{B}^N = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle, & a_3 > 0, b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle, & a_3 < 0, b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle, & a_3 < 0, b_3 < 0 \end{cases}$

4) $\lambda \tilde{A}^N = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3; \mu_A, \omega_A, \varphi_A) \rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1; \mu_A, \omega_A, \varphi_A) \rangle, & \lambda < 0 \end{cases}$

5) $\frac{\tilde{A}^N}{\tilde{B}^N} = \begin{cases} \langle (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle, & a_3 > 0, \quad b_3 > 0 \\ \langle (\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle, & a_3 < 0, \quad b_3 > 0 \\ \langle (\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}; \mu_A \wedge \mu_b, \omega_A \vee \omega_b, \varphi_A \vee \varphi_B) \rangle, & a_3 < 0, \quad b_3 < 0 \end{cases}$

Definition 5. [14]
The ranking function for triangular neutrosophic number $\tilde{A}^N = \langle (a_1, a_2, a_3; \mu_A, \omega_A, \varphi_A) \rangle$ is defined as:

$$\mathcal{R}(\tilde{A}^N) = \frac{a_1+a_2+a_3}{9}(\mu_A + (1-\omega_A) + (1-\varphi_A)).$$

Definition 6. [14]
Suppose $\tilde{A}^N$ and $\tilde{B}^N$ be two TNNs, then:
(i) $\tilde{A}^N \leq \tilde{B}^N$ if and only if $\mathcal{R}(\tilde{A}^N) \leq \mathcal{R}(\tilde{B}^N)$.
(ii) $\tilde{A}^N < \tilde{B}^N$ if and only if $\mathcal{R}(\tilde{A}^N) < \mathcal{R}(\tilde{B}^N)$.
where $\mathcal{R}$ (.) is a ranking function.

3. PROPOSED FTNLFP METHOD

The general form of fully neutrosophic linear fractional programming (FNLFP) problem with m neutrosophic constraints and n neutrosophic variables can be written as follows:

$$\left.\begin{aligned} &Max\ (or\ Min)\ \tilde{Z}^N(x) = \frac{\tilde{p}^N(x)}{\tilde{q}^N(x)} = \frac{\tilde{c}^N \otimes \tilde{x}^N + \tilde{\alpha}^N}{\tilde{d}^N \otimes \tilde{x}^N + \tilde{\beta}^N} \\ &\quad Subject\ to: \\ &\tilde{A}^N \otimes \tilde{x}^N \le \tilde{B}^N \\ &\tilde{x}^N \ge \tilde{0}^N, \quad \tilde{q}^N(x) > 0 \end{aligned}\right\} \quad (1)$$

where $\tilde{c}^N$, $\tilde{d}^N$ are (1 × n) positive vector of TNNs, $\tilde{\alpha}^N$, $\tilde{\beta}^N$ are positive TNNs, $\tilde{A}^N$ is (m × n) positive matrix of TNNs, $\tilde{B}^N$ is (m × 1) positive column of TNNs and $\tilde{x}^N$is (n × 1) of TNNs.

Now, in order to find the neutrosophic optimal solution as well as neutrosophic optimal value of problem 1, let us introduce a new strategy described in the following steps:

Step 1: substituting $\tilde{c}^N = (\tilde{c}_j^N)_{(1\times n)}$, $\tilde{d}^N = (\tilde{d}_j^N)_{(1\times n)}$, $\tilde{x}^N = (\tilde{x}_j^N)_{(1\times n)}$, $\tilde{A}^N = (\tilde{a}_{ij}^N)_{(m\times n)}$, $\tilde{B}^N = (\tilde{b}_i^N)_{(m\times 1)}$. The problem (1) may be written as:

$$\left.\begin{aligned} &Max\ (or\ Min)\ \tilde{Z}^N(x) = \frac{\sum_{j=1}^n \tilde{c}_j^N \otimes \tilde{x}_j^N + \tilde{\alpha}^N}{\sum_{j=1}^n \tilde{d}_j^N \otimes \tilde{x}_j^N + \tilde{\beta}^N} \\ &subject\ to: \\ &\sum_{j=1}^n \tilde{a}_{ij}^N \otimes \tilde{x}_j^N \le \tilde{b}_i^N; \quad i = 1,2,\dots\dots\dots m. \\ &\tilde{x}_j^N \ge \tilde{0}^N \end{aligned}\right\} \quad (2)$$

Step 2: Represent all parameters and variables by the SVTN numbers. Then the problem (2) becomes

$$\tilde{c}_j^N = (p_j, q_j, r_j; \mu_c, \omega_c, \varphi_c),$$
$$\tilde{x}_j^N = (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x),$$
$$\tilde{d}_j^N = (e_j, f_j, g; \mu_d, \omega_d, \varphi_d)$$
$$\tilde{\alpha}_j^N = (\alpha_{j1}, \alpha_{j2}, \alpha_{j3}; \mu_{j\alpha}, \omega_{j\alpha}, \varphi_{j\alpha}),$$
$$\tilde{\beta}_j^N = (\beta_{j1}, \beta_{j2}, \beta_{j3}; \mu_{j\beta}, \omega_{j\beta}, \varphi_{j\beta}),$$
$$\tilde{a}_{ij}^N = (a_{ij}, b_{ij}, c_{ij}; \mu_a, \omega_a, \varphi_a),$$
$$\tilde{b}_i^N = (h_i, k_i, l_i; \mu_b, \omega_b, \varphi_b)$$

$$\left.\begin{aligned} &Max\ (or\ Min)\ \tilde{Z}^N(x) = \frac{\sum_{j=1}^n (p_j, q_j, r_j; \mu_c, \omega_c, \varphi_c) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) + (\alpha_{j1}, \alpha_{j2}, \alpha_{j3}; \mu_{j\alpha}, \omega_{j\alpha}, \varphi_{j\alpha})}{\sum_{j=1}^n (e_j, f_j, g; \mu_d, \omega_d, \varphi_d) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) + (\beta_{j1}, \beta_{j2}, \beta_{j3}; \mu_{j\beta}, \omega_{j\beta}, \varphi_{j\beta})} \\ &\quad subject\ to: \\ &\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_a, \omega_a, \varphi_a) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) \le \\ &\qquad (h_i, k_i, l_i; \mu_b, \omega_b, \varphi_b); \quad i = 1,2,\dots\dots\dots m. \\ &\tilde{x}_j^N \ge \tilde{0}^N \end{aligned}\right\}$$

Step 3: From the above problem, two fully neutrosophic linear programming problems (P_1 and P_2) can be constructed to determine the optimal solution as follows:

$$P_1: \quad Max\ (or\ Min)\ \tilde{Z}_1^N = \sum_{j=1}^n (p_j, q_j, r_j; \mu_c, \omega_c, \varphi_c) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) + (\alpha_{j1}, \alpha_{j2}, \alpha_{j3}; \mu_{j\alpha}, \omega_{j\alpha}, \varphi_{j\alpha})$$

Subject to:

$$\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_a, \omega_a, \varphi_a) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) \le (h_i, k_i, l_i; \mu_b, \omega_b, \varphi_b); \quad i\ 1,2,\dots\dots\dots m.$$
$$\tilde{x}_j^N \ge \tilde{0}^N$$

$$P_2: \quad Min\ (or\ Max)\ \tilde{Z}_2^N = \sum_{j=1}^n (e_j, f_j, g; \mu_d, \omega_d, \varphi_d) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) + (\beta_{j1}, \beta_{j2}, \beta_{j3}; \mu_{j\beta}, \omega_{j\beta}, \varphi_{j\beta})$$

Subject to:

$$\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_a, \omega_a, \varphi_a) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) \le (h_i, k_i, l_i; \mu_b, \omega_b, \varphi_b); \quad i\ 1,2,\dots\dots\dots m.$$
$$\tilde{x}_j^N \ge \tilde{0}^N$$

Step 4: By using the arithmetic operations of SVTN numbers as well as the ranking function, the problems P_1 and P_2 can be converted into two crisp linear programming problems (P_1' , P_2') as follows:

$$P_1': \quad Max\ (or\ Min)\ Z_1 = \mathcal{R}\left(\sum_{j=1}^n (p_j, q_j, r_j; \mu_c, \omega_c, \varphi_c) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x) + (\alpha_{j1}, \alpha_{j2}, \alpha_{j3}; \mu_{j\alpha}, \omega_{j\alpha}, \varphi_{j\alpha})\right)$$

Subject to:

$$\mathcal{R}\left(\sum_{j=1}^n (a_{ij}, b_{ij}, c_{ij}; \mu_a, \omega_a, \varphi_a) \otimes (x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x)\right) \le (h_i, k_i, l_i; \mu_b, \omega_b, \varphi_b)); \quad i\ 1,2,\dots\dots\dots m.$$
$$\tilde{x}_j^N \ge \tilde{0}^N$$

$$P_2': \ Min\,(or\ Max)\ Z_2 = \mathcal{R}\Big(\sum_{j=1}^{n} \big(e_j, f_j, g; \mu_d, \omega_d, \varphi_d\big) \otimes \big(x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x\big) + \big(\beta_{j1}, \beta_{j2}, \beta_{j3}; \mu_{j\beta}, \omega_{j\beta}, \varphi_{j\beta}\big)\Big)$$

Subject to:

$$\mathcal{R}\Big(\sum_{j=1}^{n} \big(a_{ij}, b_{ij}, c_{ij}; \mu_a, \omega_a, \varphi_a\big) \otimes \big(x_j, y_j, z_j; \mu_x, \omega_x, \varphi_x\big)\Big) \leq \big(h_i, k_i, l_i; \mu_b, \omega_b, \varphi_b\big)\Big); \quad i\ 1,2,\ldots\ldots\ldots m.$$

$$\tilde{x}_j^N \geq \tilde{0}^N$$

Step 5: Find the optimal solutions and objective values by solving crisp linear programming problems (P_1', P_2').

Step 6: Find the objective value of crisp linear fractional programming problem by

$$Max\ Z = \frac{Z_1}{Z_2}$$

4. Numerical Example

In Jamshedpur City, India, A Wooden company is the producer of three kinds of products A, B and C with profit around 8, 7 and 9 dollar per unit, respectively. However, the cost for each one unit of the above products is around 8, 9 and 6 dollar respectively. It is assumed that a fixed cost of around 1.5 dollar is added to the cost function due to expected duration through the process of production .

Suppose the raw material needed for manufacturing product A, B and C is about 4, 3 and 5 units per dollar respectively, the supply for this raw material is restricted to about 28 dollars. Man-hours per unit for the product A is about 5 hours, product B is about 3 hour and C is about 3 hour per unit for manufacturing but total Man-hour available is about 20 hours daily. Determine how many products A, B and C should be manufactured in order to maximize the total profit .

Let x_1, x_2 *and* x_3 component be the measure of A, B and C, individually to be created .

After prediction of evaluated parameters, the above issue can be defined as the following FTNLFP Problem:

$$Max\ \tilde{Z}^N = \frac{\tilde{8}^N \otimes \tilde{x}_1^N + \tilde{7}^N \otimes \tilde{x}_2^N + \tilde{9}^N \otimes \tilde{x}_3^N}{\tilde{8}^N \otimes \tilde{x}_1^N + \tilde{9}^N \otimes \tilde{x}_2^N + \tilde{6}^N \otimes \tilde{x}_3^N + \widetilde{1.5}^N}$$

Subject to

$$\tilde{4}^N \otimes \tilde{x}_1^N + \tilde{3}^N \otimes \tilde{x}_2^N + \tilde{5}^N \otimes \tilde{x}_3^N \leq \widetilde{28}^N$$

$$\tilde{5}^N \otimes \tilde{x}_1^N + \tilde{3}^N \otimes \tilde{x}_2^N + \tilde{3}^N \otimes \tilde{x}_3^N \leq \widetilde{20}^N$$

$\tilde{x}_1^N, \tilde{x}_2^N, \tilde{x}_3^N$ are nonnegative TNNs.

Here we consider,

$\tilde{8}^N$= (7, 8, 9; 0.5, 0.8, 0.3) $\tilde{7}^N$= (6, 7, 8; 0.2, 0.6, 0.5) $\tilde{9}^N$= (8, 9, 10; 0.8, 0.1, 0.4) $\tilde{6}^N$= (4, 6, 8; 0.75, 0.25, 0.1) $\widetilde{1.5}^N$= (1, 1.5, 2; 0.75, 0.5, 0.25), $\tilde{4}^N$ = $(3,4,5;\ 0.4,0.6,0.5)$ $\tilde{3}^N$= (2, 3, 4; 1, 0.25, 0.3), $\tilde{5}^N$= (4, 5, 6; 0.3, 0.4, 0.3), $\widetilde{28}^N$= (25, 28, 30; 0.4, 0.25, 0.6) $\widetilde{20}^N$= (18, 20, 22; 0.9, 0.2, 0.6)

$\tilde{x}_1^N = (x_1, y_1, z_1; 0.8, 0.2, 0.3)$

$\tilde{x}_2^N = (x_2, y_2, z_2; 0.8, 0.2, 0.3)$,

$\tilde{x}_3^N = (x_3, y_3, z_3; 0.8, 0.2, 0.3)$

By using step 2 the above problem becomes

$$Max\tilde{Z}^N = \frac{\begin{array}{c}(7,8,9;\ 0.5,0.8,0.3\ )\otimes(x_1,y_1,z_1;0.8,0.2,0.3)\\ +(6,7,8;\ 0.2,0.6,0.5)\otimes(x_2,y_2,z_2;0.8,0.2,0.3)\\ +(8,9,10;\ 0.8,0.1,0.4)\otimes(x_3,y_3,z_3;0.8,0.2,0.3)\end{array}}{\begin{array}{c}(7,8,9;\ 0.5,0.8,0.3\ )\otimes(x_1,y_1,z_1;0.8,0.2,0.3)\\ +(8,9,10;\ 0.8,0.1,0.4)\otimes(x_2,y_2,z_2;0.8,0.2,0.3)\\ +(4,6,8;\ 0.75,0.25,0.1\ )\otimes(x_3,y_3,z_3;0.8,0.2,0.3)\\ +(1,1.5,2;\ 0.75,0.5,0.25)\end{array}}$$

Subject to:

$$(3,4,5;\ 0.4,0.6,0.5)\otimes(x_1,y_1,z_1;0.8,0.2,0.3) + (2,3,4;\ 1,0.25,0.3)\otimes(x_2,y_2,z_2;0.8,0.2,0.3) + (4,5,6;\ 0.3,0.4,0.3)\otimes(x_3,y_3,z_3;0.8,0.2,0.3) \leq (25,28,30;\ 0.4,0.25,0.6)$$

$$(4,5,6;\ 0.3,0.4,0.3)\otimes(x_1,y_1,z_1;0.8,0.2,0.3) + (2,3,4;\ 1,0.25,0.3)\otimes(x_2,y_2,z_2;0.8,0.2,0.3) + (2,3,4;\ 1,0.25,0.3)\otimes(x_3,y_3,z_3;0.8,0.2,0.3) \leq (18,20,22;\ 0.9,0.2,0.6)$$

$\tilde{x}_1^N, \tilde{x}_2^N, \tilde{x}_3^N$ are nonnegative TNNs.

From the above problem, two fully triangular neutrosophic linear programming problems (P_1 and P_2) can be constructed as follows:

$$\boldsymbol{P_1}: \quad Max\ \tilde{Z}_1^N = (7,8,9;\ 0.5,0.8,0.3\ )\otimes(x_1,y_1,z_1;0.8,0.2,0.3) + (6,7,8;\ 0.2,0.6,0.5)\otimes(x_2,y_2,z_2;0.8,0.2,0.3) + (8,9,10;\ 0.8,0.1,0.4)\otimes(x_3,y_3,z_3;0.8,0.2,0.3)$$

Subject to:

$$(3,4,5;\ 0.4,0.6,0.5)\otimes(x_1,y_1,z_1;0.8,0.2,0.3) + (2,3,4;\ 1,0.25,0.3)\otimes(x_2,y_2,z_2;0.8,0.2,0.3) + (4,5,6;\ 0.3,0.4,0.3)\otimes(x_3,y_3,z_3;0.8,0.2,0.3) \leq (25,28,30;\ 0.4,0.25,0.6)$$

$$(4,5,6;\ 0.3,0.4,0.3)\otimes(x_1,y_1,z_1;0.8,0.2,0.3) + (2,3,4;\ 1,0.25,0.3)\otimes(x_2,y_2,z_2;0.8,0.2,0.3) + (2,3,4;\ 1,0.25,0.3)\otimes(x_3,y_3,z_3;0.8,0.2,0.3) \leq (18,20,22;\ 0.9,0.2,0.6)$$

$\tilde{x}_1^N, \tilde{x}_2^N, \tilde{x}_3^N$ are nonnegative TNNs.

$$P_2: \ Max\tilde{Z}_2^N = \begin{matrix}(7,8,9;\ 0.5,0.8,0.3)\otimes(x_1,y_1,z_1;0.8,0.2,0.3)\\ +(8,9,10;\ 0.8,0.1,0.4)\otimes(x_2,y_2,z_2;0.8,0.2,0.3)\\ +(4,6,8;\ 0.75,0.25,0.1)\otimes(x_3,y_3,z_3;0.8,0.2,0.3)\\ +(1,1.5,2;\ 0.75,0.5,0.25)\end{matrix}$$

Subject to:

$$(3,4,5;\ 0.4,0.6,0.5)\otimes(x_1,y_1,z_1;0.8,0.2,0.3)+$$
$$(2,3,4;\ 1,0.25,0.3)\otimes(x_2,y_2,z_2;0.8,0.2,0.3)+$$
$$(4,5,6;\ 0.3,0.4,0.3)\otimes(x_3,y_3,z_3;0.8,0.2,0.3)\le(25,28,30;\ 0.4,0.25,0.6)$$
$$(4,5,6;\ 0.3,0.4,0.3)\otimes(x_1,y_1,z_1;0.8,0.2,0.3)+$$
$$(2,3,4;\ 1,0.25,0.3)\otimes(x_2,y_2,z_2;0.8,0.2,0.3)+$$
$$(2,3,4;\ 1,0.25,0.3)\otimes(x_3,y_3,z_3;0.8,0.2,0.3)\le(18,20,22;\ 0.9,0.2,0.6)$$

$\tilde{x}_1^N, \tilde{x}_2^N, \tilde{x}_3^N$ are nonnegative TNNs.

By using the arithmetic operations of TNNs as well as the ranking function, the problems P_1 and P_2 can be converted into two linear programming problems (P_1', P_2') and solved as follows:

$$P_1': \ Max\ Z_1 = 1.08x_1 + 1.24y_1 + 1.39z_1 + 0.73x_2 + 0.85y_2 + 0.97z_2 + 1.95x_3 + 2.19y_3 + 2.44z_3$$

Subject to:

$$0.43x_1 + 0.57y_1 + 0.72z_1 + 0.5x_2 + 0.75y_2 + z_2 + 0.48x_3 + 0.61y_3 + 0.73z_3 \le 14.29$$
$$0.48x_1 + 0.61y_1 + 0.73z_1 + 0.5x_2 + 0.75y_2 + z_2 + 0.5x_3 + 0.75y_3 + z_3 \le 14$$
$$x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3 \ge 0$$

Now the problem P_1' is solved by simplex method and the optimal solution is

$x_1=0,\ y_1=0,\ z_1=0,\ x_2=0,\ y_2=0,\ z_2=0,\ x_3=28,\ y_3=0,\ z_3=0$

And the objective value is $Z_1 = 54.6$.

$$P_2': \ Max\ Z_2 = 1.08x_1 + 1.24y_1 + 1.39z_1 + 1.95x_2 + 2.19y_2 + 2.44z_2 + 0.97x_3 + 1.46y_3 + 1.95z_3 + 1$$

Subject to:

$$0.43x_1 + 0.57y_1 + 0.72z_1 + 0.5x_2 + 0.75y_2 + z_2 + 0.48x_3 + 0.61y_3 + 0.73z_3 \le 14.29$$
$$0.48x_1 + 0.61y_1 + 0.73z_1 + 0.5x_2 + 0.75y_2 + z_2 + 0.5x_3 + 0.75y_3 + z_3 \le 14$$
$$x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3 \ge 0$$

Now the problem P_2' is solved by simplex method and the optimal solution is

$x_1=0,\ y_1=0,\ z_1=0,\ x_2=12.99,\ y_2=0,\ z_2=0,\ x_3=14.95,\ y_3=0,\ z_3=0$

And the objective value is $Z_2 = 39.832$.

Finally, the objective value of crisp linear fractional programming problem is

$$Max\ Z = \frac{54.6}{39.832} = 1.37$$

By comparing the results of the proposed method with existing methods [3, 10] based on ranking function, we can conclude that our result is more effective, because:

$Z_{(\text{prposed method})} = 1.37 > Z_{(\text{existing method[10]})} = 1.16 > Z_{(\text{existing method[3]})} = 1$

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5. CONCLUSIONS

In this study, we introduce the neutrosophic linear fractional programming (NLFP) problem and propose a novel model to find the neutrosophic optimal solution of the linear fractional programming problem in neutrosophic environment where all the parameters as well as decision variables are triangular neutrosophic numbers. In our proposed method, FTNLFP problem is converted into a crisp LP problem by using ranking function and solve it. Finally, from the results obtained and compared with the results of existing methods, it can be concluded that our model is more efficient and convenient.

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